

UNCLASSIFIED

## FINAL REPORT

## FILTERED BINARY PROCESSES

## (a) Statement of Work

The Contractor shall research the development, modification, refinement and integration of methods for determining the probability density functions of filtered binary processes; and study any properties which may have application to general random processes which may be suggested by the studies of filtered binary processes. The contractor shall investigate the following: 1. Ways of determining the moments  $A(y)$  and  $b(y)$  of the extended Fokker-Planck equation, 2. Further refinements of Pearson-type approximations, 3. Monte Carlo simulations for higher-order filtering, 4. Proof of the conjecture that the product of the second conditional moment and the system probability density function  $B(y)p(y)$  is proportional to the level crossing rate for a general random process.

## (b) Status of Research Effort

One paper supported under this contract is to appear in print in the IEEE Transactions on Information Theory in July 1989 and is listed as paper No. 12 in the next section. Another, No. 13 has been submitted and work is in progress on a third, No. 14. Both of these will appear in the Physical Review A.

In extending the results from paper No. 9, "Approximating Distributions from Moments," to the nonsymmetric case and infinite intervals, it was necessary to state and prove an "Equivalence Theorem." The proof of this theorem was more involved than originally anticipated. The theorem says, in brief, that if "too many" terms are taken in the numerator and denominator polynomials that arises in using Pearson-type approximations, that the "extra" terms cannot have any effect if they are not a part of the underlying density function. This is a powerful result in that it says that if the underlying density function is exact, that the "approximate" method will yield it. We now have a satisfactory proof to this theorem and have attempted to use it to search for exact solutions in the case of the filter-limiter-filter problem and the Butterworth filter driven by the random telegraph signal.

We have begun exploring extensions of the Pearson-type methods to the problem of approximating spectral density functions from discrete samples of autocorrelation functions. The motivation for this is that there is a one-to-one correspondence between probability density functions and spectral density functions since each is non-negative and integrable. Consequently, the characteristic function of the probability density function has a one-to-one correspondence with the autocorrelation function of the power spectral density function. The characteristic function determines all the moments of the underlying density; in like fashion, the autocorrelation function will determine all moments of the underlying spectrum. Hence, we expect that methods used to approximate densities from moments will have use in approximating spectra from samples of autocorrelation functions.

We have been studying cases in which the incremental behavior of a random process behaves as a nonlinear function of the incremental time spacing for small time spacings. One such case is a nonintegral power of the spacing multiplied by the natural logarithm of the spacing. For such non-Markov processes, we have calculated the conditional moments  $A(y)$  and  $B(y)$  in the extended Fokker-Planck equation. Studies relating these moments to the average number of level crossings of the underlying process have been initiated.

(c) List of Reports and Written Publications in Technical Journals

Following is a list of papers and reports and work in progress which was supported under this contract and the one preceding it under which the research was initiated. Also listed for completion is a recently published paper, paper No. 11, which was supported by other sources.

1. R.F. Pawula and J.H. Roberts, "The Effects of Noise Correlation and Power Imbalance on Terrestrial and Satellite DPSK Channels," IEEE Transactions on Communications, COM-31, pp. 750-755, June 1983.
2. R.F. Pawula, "Offset DPSK and a Comparison of Conventional and Symmetric DPSK with Noise Correlation and Power Imbalance," Proceedings of MILCOM '83, Washington, D.C., pp. 93-98, October 1983 (this is an abridged version of paper No. 4).
3. R.F. Pawula, "Asymptotics and Error Rate Bounds for M-ary DPSK," IEEE Transactions on Communications, COM-32, pp. 93-94, Jan. 1984.
4. R.F. Pawula, "Offset DPSK and a Comparison of Conventional and Symmetric DPSK with Noise Correlation and Power Imbalance," IEEE Transactions on Communications, COM-32, pp. 233-240, March 1984.
5. R.F. Pawula, "On M-ary DPSK Transmission over Terrestrial and Satellite Channels," IEEE Transactions on Communications, COM-32, pp. 752-761, July 1984.
6. R.F. Pawula and S.O. Rice, "On Filtered Binary Processes," IEEE Transactions on Information Theory, IT-32, pp. 63-72, January 1986.
7. R.F. Pawula, "Dichotomous-Noise-Driven Oscillators," Physical Review A, 35, pp. 3102-3108, April 1987.
8. R.F. Pawula and S.O. Rice, "A Differential Equation Related to a Random Telegraph Wave Problem - Computer Calculation of Series Solution," IEEE Transactions on Information Theory, IT-33, pp. 882-888, November 1987.
9. R.F. Pawula, "Approximating Distributions from Moments," Physical Review A, 36, pp. 4996-5007, November 1987.
10. R.F. Pawula, "Level Crossings of Filtered Dichotomous Noise," Physical

Review A, 37, pp. 17311735, March 1988.

11. R.F. Pawula, "Refinements to the Theory of Error Rates for Narrow-Band Digital FM," IEEE Transactions on Communications, COM-36, pp. 509-513, April 1988.
12. R.F. Pawula and S.O. Rice, "A Note on Hansmann's 1934 Family of Distributions," IEEE Transactions on Information Theory, July 1989, to appear.
13. R.F. Pawula, "Approximating Distributions from Moments II," Physical Review A, submitted.
14. R.F. Pawula, "The extended Fokker-Planck Equation," in progress.

Any internal reports which have been listed in our previous annual reports have been omitted since they have been incorporated into the actual published papers.

(d) List of Professional Personnel Associated with Research Effort

All of the work done under this contract was done by the Principal Investigator, Dr. Robert F. Pawula. Paper No. 1 was written with the collaboration of Mr. John H. Roberts of Plessey Electronics Company, Roke Manor, Romsey, Hampshire, England. Papers No. 6, 8, and 12 were written with the collaboration of Dr. S.O. Rice of the Department of Electrical Engineering & Computer Science of the University of California at San Diego (Dr. Rice is now deceased).

We continue to correspond with Dr. M.K. Simon of the Jet Propulsion Laboratory at Caltech, Dr. Nelson Blachman of GTE Sylvania in Mountain View, California, and Dr. I. Korn of the Department of Electrical Engineering of the University of New South Wales in Australia. All of these are recognized authorities on noise, stochastic processes and communications systems and all have written texts in one or more of these areas.

(e) Interactions (Coupling Activities)

The Principal Investigator has reviewed papers for the IEEE Transactions on Communications, the IEEE Transactions on Information Theory, the IEEE Transactions on Vehicular Technology and the British IEE Journal. Although there have been no direct consultative and/or advisory activities to other agencies, reprints of our work have been requested by and sent to the Department of Health and Human Services, the Naval Research Laboratory, the U.S. Navy Postgraduate School and the Naval Ocean Systems Center.

(f) Any Other Statements

We continue to receive requests for reprints of the work supported under this contract and feel that this is an important indicator of the significance of the mathematical techniques that we have been developing and refining. So far, requests have been received from many places in the United States and from Canada, Singapore, Spain, Czechoslovakia, India, Switzerland, Hungary, Romania, East Germany, West Germany, England, the U.S.S.R., Poland, Italy, Israel, Hong Kong, Argentina, Japan and Greece.

More than 100 requests for reprints were received for Paper No. 9, "Approximating Distributions from Moments." Current signal processing techniques are more and more examining important situations in which the underlying statistics of noise processes are non-Gaussian and higher-order moments are being used to detect signals in noise. This is especially true in the case of acoustic transient detection in the ocean and the tracking of submarines. Hence, we expect to see more emphasis placed in the near future on higher-order moments.

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